

### 利用经纬度求解两点球面距离

#### Haversine formula

$$h(\theta) = \sin^2\left(\frac{\theta}{2}\right) = \frac{1 - \cos(\theta)}{2}$$

$$\text{则 } h(\theta) = h\left(\frac{d}{R}\right) = h(\Delta\beta) + \cos(\beta_1)\cos(\beta_2)h(\Delta\alpha)$$

- \$R\$表示球面半径,\$d\$表示球面距离,\$\theta\$表示两点与圆心夹角弧度\$ - \$ \$\alpha\_i\$分别表示两点经度,\$\beta\_i\$表示两点维度,\$\Delta\alpha\$表示差值\$ - \$ 公式全称应该为\$half-versine\$,即\$versine\$:  $1 - \cos(\theta)$ 的一半\$ - \$计算时可进一步化解\$ \cos(\theta) = \sin(\beta\_1)\sin(\beta\_2) + \cos(\beta\_1)\cos(\beta\_2)\cos(\Delta\alpha)\$

![在这里插入图片描述]

述]([https://img-blog.csdnimg.cn/20200402125945407.jpg?x-oss-process=image/watermark,type\\_ZmFuZ3poZW5naGVpbGk&shadow\\_10|text\\_aHR0dHMbG9hJmNzZG4ubmV0L0hmRpY3Q-size\\_160&r#70](https://img-blog.csdnimg.cn/20200402125945407.jpg?x-oss-process=image/watermark,type_ZmFuZ3poZW5naGVpbGk&shadow_10|text_aHR0dHMbG9hJmNzZG4ubmV0L0hmRpY3Q-size_160&r#70))

这里求\$\overset{\frown}{AB}\$显然求得\$|AB|\$即可

以\$OEF\$为例\$|\angle OEF|=\Delta\alpha, |EF|=2\sin\left(\frac{\Delta\alpha}{2}\right)R\$同理利用维度\$|AC|=2\sin\left(\frac{\Delta\beta}{2}\right)R\$

而对于\$|BC|,|AD|\$作\$AG\perp OE,BH\perp OE\$可得\$|AD|=2\sin\left(\frac{\Delta\alpha}{2}\right)(|OE|\cos(\angle AOG))=2\sin\left(\frac{\Delta\alpha}{2}\right)R\cos(\beta\_1)\$

而四边形\$ACBD\$为等腰梯形形

$$CH=\frac{BC-AD}{2}, AB^2=BH^2+AH^2=(BC-CH)^2+AC^2-CH^2=AC^2+BC^2-2AC\cdot CH$$

$$|AB|^2=4\sin^2\left(\frac{\Delta\beta}{2}\right)R^2+4\sin^2\left(\frac{\Delta\alpha}{2}\right)\cos(\beta_1)\cos(\beta_2)R^2$$

$$\text{而要求解的 } \theta = \angle AOB, |AB|^2 = 4\sin^2\left(\frac{\theta}{2}\right)R^2$$

得到目标公式\$ h(\theta) = h(\Delta\beta) + \cos(\beta\_1)\cos(\beta\_2)h(\Delta\alpha), \overset{\frown}{AB} = d = R\theta \$

进一步化解\$ 1 - \cos(\theta) = 1 - \cos(\Delta\alpha) + \cos(\beta\_1)\cos(\beta\_2)(1 - \cos(\Delta\alpha)) \$

$$\cos(\Delta\alpha) = \cos(\beta_1)\cos(\beta_2) + \sin(\beta_1)\sin(\beta_2)$$

$$\text{可得 } \cos(\theta) = \sin(\beta_1)\sin(\beta_2) + \cos(\beta_1)\cos(\beta_2)\cos(\Delta\alpha)$$

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