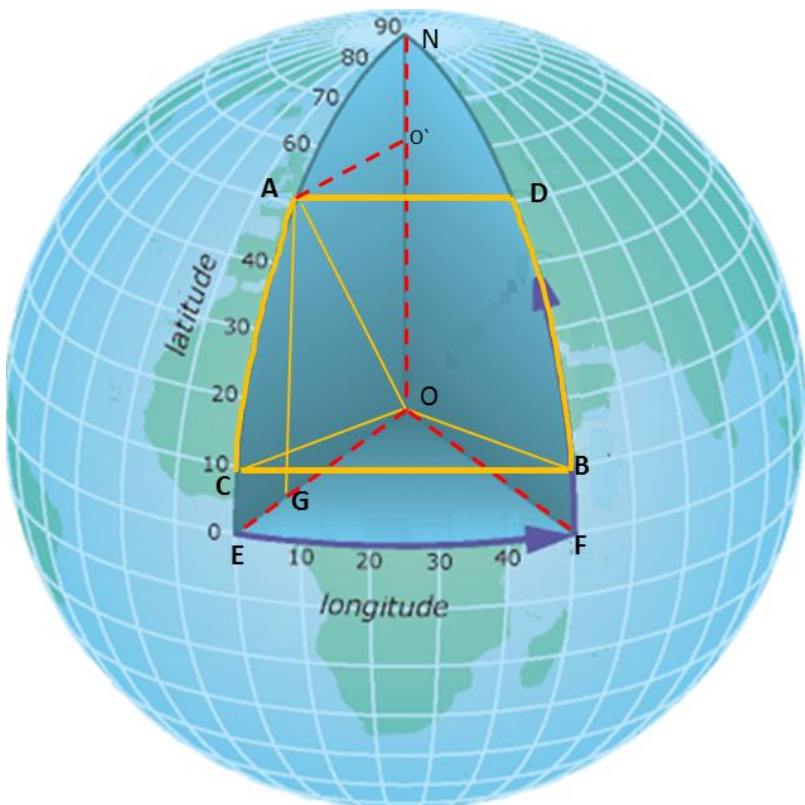


Haversine formula

$$h(\theta) = \sin^2\left(\frac{\theta}{2}\right) = \frac{1 - \cos(\theta)}{2}$$

$$\text{则 } h(\theta) = h\left(\frac{d}{R}\right) = h(\Delta \beta) + \cos(\beta_1)\cos(\beta_2)h(\Delta \alpha)$$

- R 表示球面半径, d 表示球面距离, θ 表示两点与圆心夹角弧度 - α_i 分别表示两点经度, β_i 表示两点纬度, Δ 表示差值 - 公式全称应该为 half-versine ,即 versine : $1 - \cos(\theta)$ 的一半 - 计算时可进一步化解 $\cos(\theta) = \sin(\beta_1)\sin(\beta_2) + \cos(\beta_1)\cos(\beta_2)\cos(\Delta \alpha)$



这里求 $\overset{\frown}{AB}$ 显然求得 $|AB|$ 即可

以 OE 为例 $\angle OEF = \Delta \alpha, |EF| = 2\sin\left(\frac{\Delta \alpha}{2}\right)R$ 同理利用纬度 $|AC| = 2\sin\left(\frac{\Delta \beta}{2}\right)R$

而对于 $|BC|, |AD|$ 作 $AG \perp OE, BH \perp OE$ 可得 $|AD| = 2\sin\left(\frac{\Delta \alpha}{2}\right)(|OE|\cos(\angle AOG)) = 2\sin\left(\frac{\Delta \alpha}{2}\right)R\cos(\beta_1)$

而四边形 $ACBD$ 为等腰梯形形

$$CH = \frac{BC - AD}{2}, AB^2 = BH^2 + AH^2 = (BC - CH)^2 + AC^2 - CH^2 = AC^2 + BC \cdot AD$$

$$|AB|^2 = 4\sin^2\left(\frac{\Delta \beta}{2}\right)R^2 + 4\sin^2\left(\frac{\Delta \alpha}{2}\right)\cos(\beta_1)\cos(\beta_2)R^2$$

而要求解的 $\theta = \angle AOB, |AB|^2 = 4\sin^2\left(\frac{\theta}{2}\right)R^2$

得到目标公式 $h(\theta) = h(\Delta \beta) + \cos(\beta_1)\cos(\beta_2)h(\Delta \alpha), \overset{\frown}{AB} = d = R\theta$

进一步化解 $1 - \cos(\theta) = 1 - \cos(\Delta \beta) + \cos(\beta_1)\cos(\beta_2)(1 - \cos(\Delta \alpha))$

$\cos(\Delta \beta) = \cos(\beta_1)\cos(\beta_2) + \sin(\beta_1)\sin(\beta_2)$

可得 $\cos(\theta) = \sin(\beta_1)\sin(\beta_2) + \cos(\beta_1)\cos(\beta_2)\cos(\Delta \alpha)$

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