子集卷积

简介

一般我们有如下一类的状压dp方程，如$dp[i]=\sum dp[j]*w[k]$ (i,j,k满足$j\lor k=i,j \land k=0$这里符号表示按位与和按位或。

如果暴力枚举位的子集，那么效率是3^n的，难以承受。

实际上这个已经很接近一个FWT卷积的形式了，只不过是还要$j\land k=0$罢了。

我们改变这个条件为j中1的个数+k中1的个数=i中1的个数，那么当我们为dp增加一个“1的个数”的维度时，问题迎刃而解 $$ dp[cnt_i][i]=\sum_{(j|k)==i}dp[cnt_j][j]*w[cnt_i-cnt_j][k] $$ 注意这里cnt_i表示1的个数，或者说子集中的物品数目。这里cnt_i和i的二进制1的个数如果不等，这个dp或者w值会置为0。此时只要我们从小到大枚举cnt来做FWT就可以得到答案了，实际操作过程中，所有的dp都是点值形式，因此得到新的$dp[cnt_i]$只需要做cnt_i次对位乘；最后，再将所有的dp逆FWT变换回原值。

虽然牺牲了一定空间，但是时间被优化到了n次FWT+n^2次对位乘法的复杂度$O((2^n*n)*n+n^2*2^n)=O(n^2*2^n)$

例题

模板题https://ac.nowcoder.com/acm/contest/5157/D

很容易从题目的形式看出来实际上就是对四个数列求三重卷积，第一重是$i|j$的子集卷积，第二重$(i|j)+k$的FFT/NTT第三重是$((i|j)+k)\otimes h$的FWT的异或卷积。

```
#include <bits/stdc++.h>

#define N 262144

using namespace std;

const int mod = 998244353, inv2 = 499122177;

int n;
int rev[N], lim, hib;
int A[N], B[N], C[N], D[N], popc[N];
int f[20][N], g[20][N], h[20][N];

inline int Add(int u, int v) { return (u += v) >= mod ? u - mod : u; }

inline void Inc(int &u, int v) { if ((u += v) >= mod) u -= mod; }

inline int fpm(int x, int y) {
    int r = 1;
    while (y) {
        if (y & 1) r = 1LL * x * r % mod;
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        x = 1LL * x * x % mod, y >>= 1;
    }
    return r;
}

inline int read() {
    int x = 0;
    char ch = getchar();
    while (!isdigit(ch)) ch = getchar();
    while (isdigit(ch)) x = x * 10 + ch - '0', ch = getchar();
    return x;
}

void getrev(int len) {
    lim = 1, hib = -1;
    while (lim < len) lim <<= 1, ++hib;
    for (int i = 0; i < lim; ++i)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << hib);
}

void fwtOr(int *a, bool type) {
    for (int mid = 1; mid < lim; mid <<= 1)
        for (int i = 0; i < lim; i += (mid << 1))
            for (int j = 0; j < mid; ++j)
                if (type) Inc(a[i + mid + j], a[i + j]);
                else Inc(a[i + mid + j], mod - a[i + j]);
}

void fwtXor(int *a, bool type) {
    static int x, y;
    for (int mid = 1; mid < n; mid <<= 1)
        for (int len = mid << 1, i = 0; i < n; i += len)
            for (int j = 0; j < mid; ++j) {
                x = a[i + j], y = a[i + mid + j];
                a[i + j] = Add(x, y), a[i + mid + j] = Add(x, mod - y);
                if (!type)
                    a[i + j] = 1LL * inv2 * a[i + j] % mod,
                            a[i + mid + j] = 1LL * inv2 * a[i + mid + j] %
mod;
            }
}

void NTT(int *a, bool type) {
    for (int i = 0; i < lim; ++i)
        if (i < rev[i])
            swap(a[i], a[rev[i]]);
    static int x, y;
    for (int mid = 1; mid < lim; mid <<= 1) {
        int len = mid << 1, wn = fpm(3, (mod - 1) / len);
        for (int i = 0; i < lim; i += len)
```

```
            for (int j = 0, w = 1; j < mid; ++j, w = 1LL * w * wn % mod) {
                x = a[i + j], y = 1LL * w * a[i + mid + j] % mod;
                a[i + j] = Add(x, y), a[i + mid + j] = Add(x, mod - y);
            }
    }
    if (!type) {
        reverse(a + 1, a + lim);
        int inv = fpm(lim, mod - 2);
        for (int i = 0; i < lim; ++i)
            a[i] = 1LL * inv * a[i] % mod;
    }
}

int main() {
    n = read(), ++n;
    getrev(n + n - 1);
    for (int i = 0; i < lim; ++i) popc[i] = popc[i >> 1] + (i & 1);
    for (int i = 0; i < n; ++i) A[i] = read(), f[popc[i]][i] = A[i];
    for (int i = 0; i < n; ++i) B[i] = read(), g[popc[i]][i] = B[i];
    for (int i = 0; i < n; ++i) C[i] = read();
    for (int i = 0; i < n; ++i) D[i] = read();

    for (int i = 0; i <= 17; ++i)
        fwtOr(f[i], true), fwtOr(g[i], true);
    for (int sa = 0; sa <= 17; ++sa)
        for (int sb = 0; sb + sa <= 17; ++sb)
            for (int i = 0; i < lim; ++i)
                h[sa + sb][i] = (h[sa + sb][i] + 1LL * f[sa][i] * g[sb][i])
% mod;
    for (int i = 0; i <= 17; ++i)
        fwtOr(h[i], false);
    for (int i = 0; i < lim; ++i)
        A[i] = h[popc[i]][i];

    NTT(A, true), NTT(C, true);
    for (int i = 0; i < lim; ++i)
        A[i] = 1LL * A[i] * C[i] % mod;
    NTT(A, false);

    fwtXor(A, true), fwtXor(D, true);
    for (int i = 0; i < lim; ++i)
        A[i] = 1LL * A[i] * D[i] % mod;
    fwtXor(A, false);

    int Q = read();
    while (Q--) printf("%d\n", A[read()]);
    return 0;
}
```

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