暑假题目汇总

CF809E

题意

给出一棵$n(2 \le n \le 2 \times 10^5)$个节点的树，边权为$1$。给定一个$1$到$n$的排列$a_i$。设$\operatorname{dist}(i,j)$为树上两点间距离，求$$\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n} \varphi(a_i \cdot a_j) \operatorname{dist}(i,j)\pmod{10^9+7}$$

题解

因为a_i是1到n的排列，所以我们可以设$p_{a_i}=i$。同时有以下结论$$\varphi(nm)=\frac{\varphi(n)\varphi(m)\gcd(n,m)}{\varphi(\gcd(n,m))}$$ 因此扔掉前面的分母$n(n-1)$。原式转化为$$\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\varphi(i)\varphi(j)\gcd(i,j)\operatorname{dist}(p_i, p_j)}{\varphi(\gcd(i,j))} $$ 开始反演 $$=\sum_{d=1}^{n}\frac{d}{\varphi(d)}\sum_{i=1}^{\left\lfloor \frac{n}{d} \right\rfloor}\sum_{j=1}^{\left\lfloor \frac{n}{d} \right\rfloor} \varphi(id)\varphi(jd)\operatorname{dist}(p_{id}, p_{jd}) [\gcd(i,j)=1] $$

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