

暑假题目汇总

CF809E

题意

给出一棵 $n(2 \leq n \leq 2 \times 10^5)$ 个节点的树，边权为 1 。给定一个 1 到 n 的排列 a_i 。设 $\text{dist}(i, j)$ 为树上两点间距离，求 $\frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j=1}^n \varphi(a_i \cdot a_j) \text{dist}(i, j) \pmod{10^9+7}$

题解

因为 a_i 是 1 到 n 的排列，所以我们可以设 $p_{a_i} = i$ 。同时有以下结论 $\varphi(nm) = \frac{\varphi(n)\varphi(m)\text{gcd}(n,m)}{\varphi(\text{gcd}(n,m))}$ 。因此扔掉前面的分母 $n(n-1)$ 原式转化为 $\sum_{i=1}^n \sum_{j=1}^n \frac{\varphi(i)\varphi(j)\text{gcd}(i,j)\text{dist}(p_i, p_j)}{\varphi(\text{gcd}(i,j))}$ 。开始反演，枚举 $d = \text{gcd}(i, j)$ $\sum_{d=1}^n \frac{d}{\varphi(d)} \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} \sum_{j=1}^{\lfloor \frac{n}{d} \rfloor} \varphi(id)\varphi(jd)\text{dist}(p_{id}, p_{jd}) [\text{gcd}(i, j) = 1]$ 。套用 $\epsilon = \mu * 1$ $\sum_{d=1}^n \frac{d}{\varphi(d)} \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} \sum_{j=1}^{\lfloor \frac{n}{d} \rfloor} \varphi(id)\varphi(jd)\text{dist}(p_{id}, p_{jd}) \sum_{p|\text{gcd}(i, j)} \mu(p)$ 。枚举 p $\sum_{d=1}^n \frac{d}{\varphi(d)} \sum_{p=1}^{\lfloor \frac{n}{d} \rfloor} \mu(p) \sum_{i=1}^{\lfloor \frac{n}{dp} \rfloor} \sum_{j=1}^{\lfloor \frac{n}{dp} \rfloor} \varphi(idp)\varphi(jdp)\text{dist}(p_{idp}, p_{jdp})$ 。枚举 $T = dp$ $\sum_{T=1}^n \sum_{i=1}^{\lfloor \frac{n}{T} \rfloor} \sum_{j=1}^{\lfloor \frac{n}{T} \rfloor} \varphi(iT)\varphi(jT)\text{dist}(p_{iT}, p_{jT}) \sum_{d|T} \frac{\mu(\frac{T}{d})}{\varphi(d)}$

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