2025/11/04 09:33 1/2 Contest Info

## **B. Lady Layton and Stone Game**

**题目大意**:有若干堆石子,每次可以取若干堆合并在一起,代价为合并后堆的大小。但是,堆数必须在 \$L\$ 和 \$R\$ 之间。求最小代价。

**题解**:显然每次合并时都会取最小的那几堆。假设两次相邻的合并分别取了 \$x\$ 堆和 \$y\$ 堆,若 \$x>y\$ 或 \$x\le y\$ 但是 \$x-1\$ 和 \$y+1\$ 合法,那么可以证明交换 \$x,y\$ 或者 \$x-1,y+1\$ 这样合并一定更优:

- 若 \$\sum\_{i=1}^{x}a\_{i}\le a\_{x+y}\$□那么前者代价为
  \$\sum\_{i=1}^{x}a\_{i}+\sum\_{i=1}^{x+y-1}a\_{i}\$□后者代价为
  \$\sum\_{i=1}^{x-1}a\_{i}+\sum\_{i=1}^{x+y-1}a\_{i}\$□此后两个序列变得相同。因此
  \$x-1,y+1\$ 更优。
- 否则,若\$\sum\_{i=1}^{x-1}a\_{i}\le a\_{x+y}\$□那么前者合并再次的代价为\$\sum\_{i=1}^{x}a\_{i}+\sum\_{i=x+1}^{x+y}a\_{i}\$□后者合并两次代价为\$\sum\_{i=1}^{x-1}a\_{i}+\sum\_{i=1}^{x+y-1}a\_{i}\$□后者代价更小。而合并完成之后,前者得到了\$\sum\_{i=1}^{x}a\_{i}\$ 与\$\sum\_{i=x+1}^{x+y}a\_{i}\$□后者得到了\$\sum\_{i=1}^{x}a\_{i}\$ 与\$a\_{x+y}\$□由于\$a\_{x+y}<\sum\_{i=1}^{x}a\_{i}\$□且\$a\_{x+y}\le\sum\_{i=x+1}^{x+y}a\_{i}\$□因此后者的序列前缀和总是小于前者。可以证明在这一前提下,在任意的合并树下,后者的代价总比前者要优。因此\$x-1,y+1\$ 更优。</li>
- 否则,即\$\sum\_{i=1}^{x-1}a\_{i}>a\_{x+y}\$□那么前者合并再次的代价为\$\sum\_{i=1}^{x}a\_{i}+\sum\_{i=x+1}^{x+y}a\_{i}\$□后者合并两次代价为\$\sum\_{i=1}^{x-1}a\_{i}+\sum\_{i=x}^{x+y}a\_{i}\$□两者相同。而合并完成之后,前者得到了\$\sum\_{i=1}^{x}a\_{i}\$ 与\$\sum\_{i=x+1}^{x+y}a\_{i}\$□后者得到了\$\sum\_{i=1}^{x-1}a\_{i}\$ 与\$\sum\_{i=x}^{x+y}a\_{i}\$□苦\$x\le y\$□显然\$\sum\_{i=1}^{x-1}a\_{i}\le\sum\_{i=1}^{x}a\_{i}\$ 且\$\sum\_{i=1}^{x-1}a\_{i}\le\sum\_{i=x+1}^{x+y}a\_{i}\$□与上一种情况相同。而若\$x>y\$□考虑交换\$x,y\$□并且不妨钦定第二种情况先合并成\$\sum\_{i=1}^{y}a\_{i}\$ 与\$\sum\_{i=1}^{x+y}a\_{i}\$\$□\$\$

因此合并的过程一定可以表示成 \$L,L,L,\ldots,L,t,R,R,R,\ldots,R\$□考虑两个序列 \$A\$ 和 \$B\$□其中 \$B\$ 的 \$R\$ 更多/\$L\$ 更少。显然可以从 \$A\$ 通过一些 \$-1/+1\$ 操作得到 \$B\$□\$B\$ 更优。

确定序列后,可以用一个队列直接暴力模拟,老题了。

## K. Game on a Circle

**题目大意**:有 \$n\$ 颗石子排成一个环,从石子 \$1\$ 开始遍历,每遇到一颗石子,有 \$p\$ 的概率把它删除。 对于石子 \$c\$□问它第 \$1,2,\ldots,n\$ 个被删除的概率。

**题解**:先讲一个等下要用的式子。令 \$x=(1-p)^{t}\$□那么

 $\label{thm:condition} $$ \left(1-p)^{e}\right)^{t=0}^{t$ 

找到一个很妙的做法。考虑容斥,设 \$f {i}\$ 表示 \$c\$ 后面恰有 \$i\$ 个石子的概率,那么

 $\label{topicond} $$ \left[ i\right=\sum_{|T|\leq i,T\in\{1,2,\cdot\}}^{|T|-i}_{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|\cdot}^{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}^{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i}_{|T|-i$ 

其中 \$g {T}\$ 表示 \$T\$ 中石子都在 \$c\$ 之后的概率。

 $update: \\ 2021/04/09 \\ 2020-2021: teams: intrepids word: 2020-hdu-multi-3 \ https://wiki.cvbbacm.com/doku.php?id=2020-2021: teams: intrepids word: 2020-hd$ 

那么我们只需要考虑  $T_{cup}{c}$  这个集合,甚至只需要关心  $T_{cup}$  中有几个在 [1,c-1] 同有几个在 [c+1,n] 即可。

设有 \$i\$ 个在 \$[1,c-1]\$□\$j\$ 个在 \$[c+1,n]\$□那么这样的 \$T\$ 有 \${c-1\choose i}{n-c\choose j}\$ 个,合法的概率为(设 \$c\$ 在第 \$t\$ 轮删除):

 $\label{thm:condition} $$ \left((1-p)^{t+1}\right)^{i}\left((1-p)^{t+1}\right)^{i}\left((1-p)^{t}\right)^{j+1}p\\ = &\left(p(1-p)^{i}\right)^{i+j+1} \end{aligned} $$$ 

显然可以卷积求出 \$g\$□然后再卷积求出 \$f\$ 即可。

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