设 $f(a,b,c,n,t_{1},t_{2})=\sum_{i=0}^{n}i^{t_{1}}\lfloor\frac{ai+b}{c}\rfloor^{t_{2}}$□求解这一函数值的算法因类似于欧几里得算法而得名类欧几里得算法。为了方便，这里仅讨论 6 个参数均为非负整数的情况。另外我们定义 $0^{0}=1$。现将该算法描述如下：

定义 $m=\lfloor\frac{an+b}{c}\rfloor$□$g_{t}(x)=(x+1)^{t}-x^{t}=\sum_{i=0}^{t-1}g_{ti}x^{i}$□$h_{t}(x)=\sum_{i=1}^{x}i^{t}=\sum_{i=0}^{t+1}h_{ti}x^{i}$□

首先描述几种特殊情况：

- 若 $t_{2}=0$□那么有：

$$ \begin{aligned} 原式&=\sum_{i=0}^{n}i^{t_{1}}\\ &=h_{t_{1}}(n)+[t_{1}=0] \end{aligned} $$

- 若 $m=0$□那么显然有：

$$ 原式=0 $$

- 若 $a=0$□那么有：

$$ \begin{aligned} 原式&=\sum_{i=0}^{n}i^{t_{1}}\lfloor\frac{b}{c}\rfloor^{t_{2}}\\ &=\lfloor\frac{b}{c}\rfloor^{t_{2}}(h_{t_{1}}(n)+[t_{1}=0]) \end{aligned} $$

- 若 $a\ge c$ 或 $b\ge c$□那么有：

$$ \begin{aligned} 原式&=\sum_{i=0}^{n}i^{t_{1}}(\lfloor\frac{(a\%c)i+(b\%c)}{c}\rfloor+\lfloor\frac{a}{c}\rfloor i+\lfloor\frac{b}{c}\rfloor)^{t_{2}}\\ &=\sum_{i=0}^{n}i^{t_{1}}\sum_{u_{1}+u_{2}+u_{3}=t_{2}}{t_{2}\choose u_{1},u_{2},u_{3}}(\lfloor\frac{(a\%c)i+(b\%c)}{c}\rfloor)^{u_{1}}(\lfloor\frac{a}{c}\rfloor i)^{u_{2}}(\lfloor\frac{b}{c}\rfloor)^{u_{3}}\\ &=\sum_{u_{1}+u_{2}+u_{3}=t_{2}}{t_{2}\choose u_{1},u_{2},u_{3}}(\lfloor\frac{a}{c}\rfloor)^{u_{2}}(\lfloor\frac{b}{c}\rfloor)^{u_{3}}\sum_{i=0}^{n}i^{t_{1}+u_{2}}(\lfloor\frac{(a\%c)i+(b\%c)}{c}\rfloor)^{u_{1}}\\ &=\sum_{u_{1}+u_{2}+u_{3}=t_{2}}{t_{2}\choose u_{1},u_{2},u_{3}}(\lfloor\frac{a}{c}\rfloor)^{u_{2}}(\lfloor\frac{b}{c}\rfloor)^{u_{3}}f(a\%c,b\%c,c,n,t_{1}+u_{2},u_{1}) \end{aligned} $$

接下来讨论一般的 $0\le a,b<c$ 的情况□ $$ \begin{aligned} 原式&=\sum_{i=0}^{n}i^{t_{1}}\sum_{j=1}^{m}g_{t_{2}}(j-1)[j\le\lfloor\frac{ai+b}{c}\rfloor]\\ &=\sum_{i=0}^{n}i^{t_{1}}\sum_{j=0}^{m-1}g_{t_{2}}(j)[j+1\le\lfloor\frac{ai+b}{c}\rfloor]\\ &=\sum_{j=0}^{m-1}g_{t_{2}}(j)\sum_{i=0}^{n}i^{t_{1}}[cj+c-b\le ai] \end{aligned} $$ 显然不论如何□$i=0$ 时 $cj+c-b\ge c-b>0\ge ai$□故 $cj+c-b\le ai$ 永远为假，因此□ $$ \begin{aligned} 原式&=\sum_{j=0}^{m-1}g_{t_{2}}(j)\sum_{i=1}^{n}i^{t_{1}}[cj+c-b\le ai]\\ &=\sum_{j=0}^{m-1}g_{t_{2}}(j)\sum_{i=1}^{n}i^{t_{1}}[cj+c-b-1<ai]\\ &=\sum_{j=0}^{m-1}g_{t_{2}}(j)\sum_{i=1}^{n}i^{t_{1}}(1-[cj+c-b-1\ge ai])\\ &=m^{t_{2}}h_{t_{1}}(n)-\sum_{j=0}^{m-1}g_{t_{2}}(j)\sum_{i=1}^{n}i^{t_{1}}[i\le\lfloor\frac{cj+c-b-1}{a}\rfloor] \end{aligned} $$

下面我们证明 $\lfloor\frac{cj+c-b-1}{a}\rfloor\le n$□ $$ \begin{aligned} \lfloor\frac{cj+c-b-1}{a}\rfloor&\le\lfloor\frac{c(m-1)+c-b-1}{a}\rfloor\\ &=\lfloor\frac{mc-b-1}{a}\rfloor

\end{aligned} $$ 假设 $\lfloor\frac{mc-b-1}{a}\rfloor>n$□那么□ $$ \begin{aligned} mc-b-1&>na\\ na+b+1&<mc\\ \end{aligned} $$ 而 $\lfloor\frac{na+b}{c}\rfloor=m$□故 $na+b\ge mc$□矛盾。故 $\lfloor\frac{cj+c-b-1}{a}\rfloor\le n$□故有□ $$ \begin{aligned} 原式&=m^{t_{2}}h_{t_{1}}(n)-\sum_{j=0}^{m-1}g_{t_{2}}(j)\sum_{i=1}^{\lfloor\frac{cj+c-b-1}{a}\rfloor}i^{t_{1}}\\ &=m^{t_{2}}h_{t_{1}}(n)-\sum_{j=0}^{m-1}g_{t_{2}}(j)h_{t_{1}}(\lfloor\frac{cj+c-b-1}{a}\rfloor)\\ &=m^{t_{2}}h_{t_{1}}(n)-\sum_{j=0}^{m-1}(\sum_{u=0}^{t_{2}-1}g_{t_{2}u}j^{u})\cdot(\sum_{v=0}^{t_{1}+1}h_{t_{1}v}\lfloor\frac{cj+c-b-1}{a}\rfloor^{v})\\ &=m^{t_{2}}h_{t_{1}}(n)-\sum_{u=0}^{t_{2}-1}\sum_{v=0}^{t_{1}+1}g_{t_{2}u}h_{t_{1}v}\sum_{j=0}^{m-1}j^{u}\lfloor\frac{cj+c-b-1}{a}\rfloor^{v}\\ &=m^{t_{2}}h_{t_{1}}(n)-\sum_{u=0}^{t_{2}-1}\sum_{v=0}^{t_{1}+1}g_{t_{2}u}h_{t_{1}v}f(c,c-b-1,a,m-1,u,v) \end{aligned} $$ 易见 (a,c) 在一次递归后变为 $(c,a\%c)$□与欧几里得算法相同。故总复杂度为 $\mathcal{O}(\log\max\{a,c\}(t_{1}+t_{2})^{4})$□