

欧拉函数

定义

$1 \sim N$ 中与 N 互质的数的个数叫欧拉函数, 记为 $\varphi(N)$

对 N 分解质因数 $N = p_1^{c_1} \times p_2^{c_2} \times \cdots \times p_k^{c_k}$

特别地, $\varphi(1) = 1$

性质

- $\varphi(N) = N \times (1 - \frac{1}{p_1}) \times (1 - \frac{1}{p_2}) \times \cdots \times (1 - \frac{1}{p_k})$
- 若有 $p \mid N$, 且满足 $p^2 \nmid N$, 则 $\varphi(N) = \varphi(N/p) \times p$
- 若有 $p \mid N$, 且一定不满足 $p^2 \nmid N$, 则 $\varphi(N) = \varphi(N/p) \times (p-1)$
- $\forall N > 1, 1 \sim N$ 中与 N 互质的数的和为 $N \times \varphi(N) / 2$
- 当 a, b 互质时, 有 $\varphi(a \times b) = \varphi(a) \times \varphi(b)$
- 对 N 分解质因数 $N = p_1^{c_1} \times p_2^{c_2} \times \cdots \times p_k^{c_k}$, 则有 $\varphi(N) = \prod_{i=1}^k \varphi(p_i^{c_i})$
- $\varphi(p^k) = p^k - p^{k-1}$
- $\sum_{d \mid n} \varphi(d) = n$
- 若正整数 a, n 互质, 则有 $a^{\varphi(n)} \equiv 1 \pmod{n}$
- 若正整数 a, n 互质, 则对于任意正整数 b , 有 $a^b \equiv a^{b \bmod \varphi(n)} \pmod{n}$
- $a, n \in \mathbb{N}$, 则 $a^b \equiv \left(\begin{array}{l} a^b \pmod{n} \& b < \varphi(n), \\ a^{b \bmod \varphi(n) + \varphi(n)} \pmod{n} \& b \geq \varphi(n). \end{array} \right. \pmod{n}$
- 若 $\gcd(m, n) = d$, 则 $\varphi(mn) = d \times \varphi(m) \times \varphi(n) / \varphi(d)$
- 若 $m \mid n$, 则 $\varphi(mn) = m \times \varphi(n)$
- 若 $m \nmid n$, 则 $\varphi(m) \mid \varphi(n)$
- $\varphi(n) = \sum_{d \mid n} d \times \mu(\frac{n}{d})$

参考资料

<https://blog.csdn.net/niiick/article/details/81347041>

<https://www.cnblogs.com/BlueHeart0621/p/11706153.html>

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