

# 欧拉函数

## 定义

$\sim N$  中与  $N$  互质的数的个数叫欧拉函数, 记为  $\varphi(N)$

对  $N$  分解质因数  $N = p_1^{c_1} \times p_2^{c_2} \times \cdots \times p_k^{c_k}$

特别地,  $\varphi(1) = 1$

## 性质

1.  $\varphi(N) = N \times (1 - \frac{1}{p_1}) \times (1 - \frac{1}{p_2}) \times \cdots \times (1 - \frac{1}{p_k})$
2. 若有  $p \mid N$ , 且满足  $p^2 \mid N$ , 则  $\varphi(N) = \varphi(N/p) \times p$
3. 若有  $p \mid N$ , 且一定不满足  $p^2 \mid N$ , 则  $\varphi(N) = \varphi(N/p) \times (p-1)$
4.  $\forall N > 1, \sum_{N \mid n} \varphi(n) = N \times \varphi(N)/2$
5. 当  $a, b$  互质时, 有  $\varphi(a \times b) = \varphi(a) \times \varphi(b)$
6. 对  $N$  分解质因数  $N = p_1^{c_1} \times p_2^{c_2} \times \cdots \times p_k^{c_k}$ , 则有  $\varphi(N) = \prod_{i=1}^k \varphi(p_i^{c_i})$
7.  $\varphi(p^k) = p^k - p^{k-1}$
8.  $\sum_{d \mid n} \varphi(d) = n$
9. 若正整数  $a, n$  互质, 则有  $a^{\varphi(n)} \equiv 1 \pmod{n}$
10. 若正整数  $a, n$  互质, 则对于任意正整数  $b$ , 有  $a^b \equiv a^b \pmod{\varphi(n)} \pmod{n}$
11.  $a, n \in N$ , 则  $a^b \equiv \left\{ \begin{array}{ll} a^b \pmod{n} & \text{if } b < \varphi(n), \\ a^b \pmod{\varphi(n) + \varphi(n)} \pmod{n} & \text{if } b \geq \varphi(n). \end{array} \right.$
12. 若  $\gcd(m, n) = d$ , 则  $\varphi(mn) = d \varphi(m) \varphi(n) / \varphi(d)$
13. 若  $m \mid n$ , 则  $\varphi(mn) = m \varphi(n)$
14. 若  $m \mid n$ , 则  $\varphi(m) \mid \varphi(n)$
15.  $\varphi(n) = \sum_{d \mid n} d \cdot \mu(\frac{n}{d})$

## 参考资料

<https://blog.csdn.net/niiick/article/details/81347041>

<https://www.cnblogs.com/BlueHeart0621/p/11706153.html>

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