

# 欧拉函数

## 定义

$1 \sim N$  中与  $N$  互质的数的个数叫欧拉函数, 记为  $\varphi(N)$

对  $N$  分解质因数  $N=p_1^{c_1} \times p_2^{c_2} \times \dots \times p_k^{c_k}$

特别地,  $\varphi(1)=1$

## 性质

- $\varphi(N)=N \times (1-\frac{1}{p_1}) \times (1-\frac{1}{p_2}) \times \dots \times (1-\frac{1}{p_k})$
- 若有  $p \mid N$ , 且满足  $p^2 \nmid N$ , 则  $\varphi(N)=\varphi(N/p) \times p$
- 若有  $p \mid N$ , 且一定不满足  $p^2 \mid N$ , 则  $\varphi(N)=\varphi(N/p) \times (p-1)$
- $\forall N > 1, 1 \sim N$  中与  $N$  互质的数的和为  $N \times \varphi(N) / 2$
- 当  $a, b$  互质时, 有  $\varphi(a \times b) = \varphi(a) \times \varphi(b)$
- 对  $N$  分解质因数  $N=p_1^{c_1} \times p_2^{c_2} \times \dots \times p_k^{c_k}$ , 则有  $\varphi(N) = \prod_{i=1}^k \varphi(p_i^{c_i})$
- $\varphi(p^k) = p^k - p^{k-1}$
- $\sum_{d \mid n} \varphi(d) = n$
- 若正整数  $a, n$  互质, 则有  $a^{\varphi(n)} \equiv 1 \pmod{n}$
- 若正整数  $a, n$  互质, 则对于任意正整数  $b$ , 有  $a^b \equiv a^{\{b \bmod \varphi(n)\} \pmod{n}}$
- $a, n \in \mathbb{N}$ , 则  $a^b \equiv \begin{cases} a^b \pmod{n}, & b < \varphi(n) \\ a^{b \bmod \varphi(n) + \varphi(n)} \pmod{n}, & b \geq \varphi(n) \end{cases}$
- 若  $\gcd(m, n) = d$ , 则  $\varphi(mn) = d \times \varphi(m) \times \varphi(n) / \varphi(d)$
- 若  $m \mid n$ , 则  $\varphi(mn) = m \times \varphi(n)$
- 若  $m \nmid n$ , 则  $\varphi(m) \mid \varphi(n)$
- $\varphi(n) = \sum_{d \mid n} d \times \mu(\frac{n}{d})$

## 参考资料

<https://blog.csdn.net/niiick/article/details/81347041>

<https://www.cnblogs.com/BlueHeart0621/p/11706153.html>

From: <https://wiki.cvbbacm.com/> - CVBB ACM Team

Permanent link: [https://wiki.cvbbacm.com/doku.php?id=2020-2021:teams:legal\\_string:%E6%AC%A7%E6%8B%89%E5%87%BD%E6%95%B0\\_igwza&rev=1593786717](https://wiki.cvbbacm.com/doku.php?id=2020-2021:teams:legal_string:%E6%AC%A7%E6%8B%89%E5%87%BD%E6%95%B0_igwza&rev=1593786717)

Last update: 2020/07/03 22:31