

[比赛链接](#)

补题情况

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C	0	0	0
D	0	0	0
E	2	0	0
H	2	0	0
J	0	0	0
M	0	0	0

题解

E. Easy Math Problem

题意

\$\$\$ \sum_{i=1}^n \sum_{j=1}^n \binom{i+j}{j} f(i,j) \backslash f(i,j) = \begin{cases} 0, & i=0 \\ a, & i=1 \\ b \times f(i-1,j) + c \times f(i-2,j), & 2 \leq i \leq j \\ d \times f(i-1,j) + e \times f(i-2,j), & i > j \end{cases} \$\$\$

题解

设

\$\$\$ A = \begin{pmatrix} b & 1 \\ c & 0 \end{pmatrix}, B = \begin{pmatrix} d & 1 \\ e & 0 \end{pmatrix} \$\$\$

不难发现

\$\$\$ (f(i+j,j), f(i+j-1,j)) = (a,0) A^{i-1} B^j \$\$\$

设 $S(i) = \sum_{j=1}^n \binom{i+j}{j} A^{i-1} B^j$ 于是有

\$\$\$ \begin{equation} \begin{split} S(i+1) &= \sum_{j=1}^n \binom{i+j+1}{j} A^i B^j \\ &= \sum_{j=1}^n \left(\binom{i+j}{j} + \binom{i+j}{j+1} \right) A^i B^j \\ &= AS(i) + \sum_{j=0}^{n-1} \binom{i+j}{j} A^{i-1} B^{j+1} \\ &= AS(i) + \left(S(i+1) + A^{i-1} B^{n+1} \right) B \end{split} \end{equation} \$\$\$ 移项，得 $S(i+1) = \left(AS(i) + A^i B^{n+1} \right) (E-B)^{-1}$

不难发现 $|E-B| = 1-d-e \neq 0$ 所以 $(E-B)$ 可逆。通过预处理上式可实现 $O(n)$ 递推。

```
const int MAXN=1e5+5,mod=998244353;
int quick_pow(int n,int k){
    int ans=1;
    while(k){
```

```
        if(k&1)ans=1LL*ans*n%mod;
        n=1LL*n*n%mod;
        k>>=1;
    }
    return ans;
}
int frac[MAXN<<1],invf[MAXN<<1];
int C(int n,int m){
    return 1LL*frac[n]*invf[m]%mod*invf[n-m]%mod;
}
void Init(){
    int n=MAXN<<1;
    frac[0]=1;
    _for(i,1,n)
        frac[i]=1LL*frac[i-1]*i%mod;
    invf[n-1]=quick_pow(frac[n-1],mod-2);
    for(int i=n-1;i;i--)
        invf[i-1]=1LL*invf[i]*i%mod;
}
struct Matrix{
    int a[2][2];
    Matrix(int a00=0,int a01=0,int a10=0,int a11=0){
        a[0][0]=a00;
        a[0][1]=a01;
        a[1][0]=a10;
        a[1][1]=a11;
    }
    Matrix operator * (const Matrix &b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
            c.a[i][j]=(1LL*a[i][0]*b.a[0][j]+1LL*a[i][1]*b.a[1][j])%mod;
        return c;
    }
    Matrix operator * (const int b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
            c.a[i][j]=1LL*a[i][j]*b%mod;
        return c;
    }
    Matrix operator + (const Matrix &b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
            c.a[i][j]=(a[i][j]+b.a[i][j])%mod;
        return c;
    }
    Matrix operator - (const Matrix &b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
            c.a[i][j]=(a[i][j]+mod-b.a[i][j])%mod;
        return c;
    }
};
```

```

    }
}A[MAXN],B[MAXN],f[MAXN];
Matrix Inv(Matrix mat){
    static int temp[2][4];
    _for(i,0,2)_for(j,0,2)
    temp[i][j]=mat.a[i][j];
    temp[0][2]=1,temp[0][3]=0;
    temp[1][2]=0,temp[1][3]=1;
    _for(i,0,2){
        int pos=-1;
        _for(j,i,2){
            if(temp[j][i]){
                pos=j;
                break;
            }
        }
        if(pos!=i)swap(temp[i],temp[pos]);
        for(int j=3;j>=i;j--)
        temp[i][j]=1LL*temp[i][j]*quick_pow(temp[i][i],mod-2)%mod;
        _for(j,0,2){
            if(j==i)continue;
            for(int k=3;k>=i;k--)
            temp[j][k]=(temp[j][k]-1LL*temp[j][i]*temp[i][k])%mod;
        }
    }
    Matrix ans;
    _for(i,0,2)_for(j,0,2)
    ans.a[i][j]=(temp[i][j+2]+mod)%mod;
    return ans;
}
void solve(){
    int
n=read_int(),a=read_int(),b=read_int(),c=read_int(),d=read_int(),e=read_int
();
    A[0]=B[0]=Matrix(1,0,0,1);
    A[1]=Matrix(b,1,c,0);
    B[1]=Matrix(d,1,e,0);
    _rep(i,1,n){
        A[i+1]=A[i]*A[1];
        B[i+1]=B[i]*B[1];
    }
    Matrix div=Inv(B[0]-B[1]);
    f[1]=Matrix();
    _rep(i,1,n)
    f[1]=f[1]+B[i]*(i+1);
    _for(i,1,n)
    f[i+1]=(A[1]*f[i]-A[i]*B[n+1]*C(i+n+1,i+1)+A[i]*B[1])*div;
    Matrix ans=Matrix();
    _rep(i,1,n)
    ans=ans+f[i];
    enter(1LL*ans.a[0][0]*a%mod);
}

```

```
}  
int main(){  
    Init();  
    int T=read_int();  
    while(T--)  
        solve();  
    return 0;  
}
```

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Last update: 2021/08/30 21:35