比赛链接

补题情况

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C	0	0	0
D	0	0	0
E	2	0	0
H	2	0	0
J	0	0	0
M	0	0	0

题解

E. Easy Math Problem

题意

$$ \sum_{i=1}^n\sum_{j=1}^n {i+j\choose j}f(i,j)\\ f(i,j)= \begin{cases} 0, &i=0\\ a, &i=1\\ b\times f(i-1,j)+c\times f(i-2,j), &2\le i\le j\\ d\times f(i-1,j)+e\times f(i-2,j), &i\gt j \end{cases} $$

题解

设

$$ A=\begin{pmatrix}b & 1 \\ c & 0\\ \end{pmatrix},B=\begin{pmatrix}d & 1 \\ e & 0\\ \end{pmatrix} $$

不难发现

$$ (f(i+j,j),f(i+j-1,j))=(a,0)A^{i-1}B^j $$

设 $S(i)=\sum_{j=1}^n {i+j\choose j}A^{i-1}B^j$于是有

$$ \begin{equation}\begin{split} S(i+1)&=\sum_{j=1}^n {i+j+1\choose j}A^iB^j \\ &=\sum_{j=1}^n \left({i+j\choose j}+{i+j\choose j+1}\right)A^iB^j\\ &=AS(i)+\sum_{j=0}^{n-1}{i+j\choose j}A^{i-1}B^jB\\ & =AS(i)+\left(S(i+1)+A^i-{i+n+1\choose i+1}A^iB^n\right)B \end{split}\end{equation} $$ 移项，得 $$ S(i+1)=\left(AS(i)+A^iB-{i+n+1\choose i+1}A^iB^{n+1}\right)(E-B)^{-1} $$

不难发现 $|E-B|=1-d-e\lt 0$所以 $(E-B)$ 可逆。通过预处理上式可实现 $O(n)$ 递推。

```
const int MAXN=1e5+5,mod=998244353;
int quick_pow(int n,int k){
    int ans=1;
    while(k){
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        if(k&1)ans=1LL*ans*n%mod;
        n=1LL*n*n%mod;
        k>>=1;
    }
    return ans;
}
int frac[MAXN<<1],invf[MAXN<<1];
int C(int n,int m){
    return 1LL*frac[n]*invf[m]%mod*invf[n-m]%mod;
}
void Init(){
    int n=MAXN<<1;
    frac[0]=1;
    _for(i,1,n)
    frac[i]=1LL*frac[i-1]*i%mod;
    invf[n-1]=quick_pow(frac[n-1],mod-2);
    for(int i=n-1;i;i--)
    invf[i-1]=1LL*invf[i]*i%mod;
}
struct Matrix{
    int a[2][2];
    Matrix(int a00=0,int a01=0,int a10=0,int a11=0){
        a[0][0]=a00;
        a[0][1]=a01;
        a[1][0]=a10;
        a[1][1]=a11;
    }
    Matrix operator * (const Matrix &b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
        c.a[i][j]=(1LL*a[i][0]*b.a[0][j]+1LL*a[i][1]*b.a[1][j])%mod;
        return c;
    }
    Matrix operator * (const int b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
        c.a[i][j]=1LL*a[i][j]*b%mod;
        return c;
    }
    Matrix operator + (const Matrix &b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
        c.a[i][j]=(a[i][j]+b.a[i][j])%mod;
        return c;
    }
    Matrix operator - (const Matrix &b)const{
        Matrix c;
        _for(i,0,2)_for(j,0,2)
        c.a[i][j]=(a[i][j]+mod-b.a[i][j])%mod;
        return c;
```

```
    }
}A[MAXN],B[MAXN],f[MAXN];
Matrix Inv(Matrix mat){
    static int temp[2][4];
    _for(i,0,2)_for(j,0,2)
    temp[i][j]=mat.a[i][j];
    temp[0][2]=1,temp[0][3]=0;
    temp[1][2]=0,temp[1][3]=1;
    _for(i,0,2){
        int pos=-1;
        _for(j,i,2){
            if(temp[j][i]){
                pos=j;
                break;
            }
        }
        if(pos!=i)swap(temp[i],temp[pos]);
        for(int j=3;j>=i;j--)
        temp[i][j]=1LL*temp[i][j]*quick_pow(temp[i][i],mod-2)%mod;
        _for(j,0,2){
            if(j==i)continue;
            for(int k=3;k>=i;k--)
            temp[j][k]=(temp[j][k]-1LL*temp[j][i]*temp[i][k])%mod;
        }
    }
    Matrix ans;
    _for(i,0,2)_for(j,0,2)
    ans.a[i][j]=(temp[i][j+2]+mod)%mod;
    return ans;
}
void solve(){
    int
n=read_int(),a=read_int(),b=read_int(),c=read_int(),d=read_int(),e=read_int
();
    A[0]=B[0]=Matrix(1,0,0,1);
    A[1]=Matrix(b,1,c,0);
    B[1]=Matrix(d,1,e,0);
    _rep(i,1,n){
        A[i+1]=A[i]*A[1];
        B[i+1]=B[i]*B[1];
    }
    Matrix div=Inv(B[0]-B[1]);
    f[1]=Matrix();
    _rep(i,1,n)
    f[1]=f[1]+B[i]*(i+1);
    _for(i,1,n)
    f[i+1]=(A[1]*f[i]-A[i]*B[n+1]*C(i+n+1,i+1)+A[i]*B[1])*div;
    Matrix ans=Matrix();
    _rep(i,1,n)
    ans=ans+f[i];
    enter(1LL*ans.a[0][0]*a%mod);
```

```
}
int main(){
    Init();
    int T=read_int();
    while(T--)
    solve();
    return 0;
}
```