

CodeChef March Challenge 2021

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An Interesting Sequence

题意

给定 T 个询问，每次询问给定 k 求

$$\sum_{i=1}^{2k} \text{gcd}(k+i^2, k+(i+1)^2)$$

题解

$$(k+i^2, k+(i+1)^2) = (k+i^2, 2i+1) = (4k+4i^2, 2i+1) = (4k+1 + (2i+1)\text{last}(2i-1), 2i+1) = (4k+1, 2i+1)$$

于是题目转化为求

$$\sum_{i=1}^{2k} \text{gcd}(4k+1, 2i+1)$$

记 $f(n) = \sum_{i=1}^n \text{gcd}(n, i)$, $g(n) = \sum_{i=1}^n \text{gcd}(n, i)[2 \mid i]$ 于是有

$$\begin{aligned} g(4k+1) + g(4k+1) &= \sum_{i=1}^{4k+1} \text{gcd}(4k+1, 2i) + \sum_{i=1}^{4k+1} \text{gcd}(4k+1, 4k+1-2i) \\ &= f(4k+1) - (4k+1) \end{aligned}$$

于是有

$$\begin{aligned} \sum_{i=1}^{2k} \text{gcd}(4k+1, 2i+1) &= f(4k+1) - 1 - g(4k+1) = f(4k+1) - 1 - \frac{f(4k+1) - (4k+1)}{2} \\ &= \frac{f(4k+1) + (4k+1)}{2} \end{aligned}$$

接下来考虑计算 f 有

$$\begin{aligned} f(n) &= \sum_{d \mid n} d \sum_{i=1}^n [\text{gcd}(n, i) = d] = \sum_{d \mid n} d \sum_{i=1}^{\frac{n}{d}} \text{gcd}(d, i) \\ &= \sum_{d \mid n} d \left[\frac{n}{d} \right] \varphi\left(\frac{n}{d}\right) \end{aligned}$$

于是可以 $O(k \log k)$ 预处理。

```
const int MAXK=1e6, MAXM=MAXK*4+5;
int prime[MAXM], phi[MAXM];
LL f[MAXM];
int main()
{
    int p_cnt=0;
    phi[1]=1;
    _for(i, 2, MAXM) {
        if (prime[i]) {
            p_cnt++;
            for (int j=i*i; j<=MAXM; j+=i)
                prime[j]=1;
        }
        phi[i]=(i-prime[i])*(p_cnt+1);
    }
}
```

```
if(!phi[i])prime[p_cnt++]=i,phi[i]=i-1;
for(int j=0;j<p_cnt&&i*prime[j]<MAXM;j++){
    if(i%prime[j]==0){
        phi[i*prime[j]]=phi[i]*prime[j];
        break;
    }
    else
        phi[i*prime[j]]=phi[i]*(prime[j]-1);
}
_for(i,1,MAXM){
    for(int j=1;i*j<MAXM;j++)
        f[i*j]+=1LL*j*phi[i];
}
int T=read_int();
while(T--){
    int k=read_int();
    enter(f[4*k+1]-1-(f[4*k+1]-(4*k+1))/2);
}
return 0;
}
```

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