CodeChef March Challenge 2021

比赛链接

An Interesting Sequence

题意

给定 T 个询问，每次询问给定 k，求

$$ \sum_{i=1}^{2k}\text{gcd}(k+i^2,k+(i+1)^2) $$

题解

$$ (k+i^2,k+(i+1)^2)=(k+i^2,2i+1)=(4k+4i^2,2i+1)=(4k+1+(2i+1)\ast(2i-1),2i+1)=(4k+1,2i+1) $$

于是题目转化为求

$$ \sum_{i=1}^{2k}\text{gcd}(4k+1,2i+1) $$

记 $f(n)=\sum_{i=1}^n\text{gcd}(n,i),g(n)=\sum_{i=1}^n\text{gcd}(n,i)[2\mid i]$，于是有

$$ g(4k+1)+g(4k+1)=\sum_{i=1}^{2k}\text{gcd}(4k+1,2i)+\sum_{i=1}^{2k}\text{gcd}(4k+1,4k+1-2i)=f(4k+1)-(4k+1) $$

于是有

$$ \sum_{i=1}^{2k}\text{gcd}(4k+1,2i+1)=f(4k+1)-1-g(4k+1)=f(4k+1)-1-\frac{f(4k+1)-(4k+1)}2 $$

接下来考虑计算 f，有

$$ f(n)=\sum_{d\mid n} d\sum_{i=1}^n[\text{gcd}(n,i)=d]=\sum_{d\mid n} d\sum_{i=1}^n[\text{gcd}(\frac nd,\frac id)=1]=\sum_{d\mid n} \varphi(\frac nd)d $$

于是可以 $O(k\log k)$ 预处理。

```
const int MAXK=1e6,MAXM=MAXK*4+5;
int prime[MAXM],phi[MAXM];
LL f[MAXM];
int main()
{
    int p_cnt=0;
    phi[1]=1;
    _for(i,2,MAXM){
```

```
        if(!phi[i])prime[p_cnt++]=i,phi[i]=i-1;
        for(int j=0;j<p_cnt&&i*prime[j]<MAXM;j++){
            if(i%prime[j]==0){
                phi[i*prime[j]]=phi[i]*prime[j];
                break;
            }
            else
            phi[i*prime[j]]=phi[i]*(prime[j]-1);
        }
    }
    _for(i,1,MAXM){
        for(int j=1;i*j<MAXM;j++)
        f[i*j]+=1LL*j*phi[i];
    }
    int T=read_int();
    while(T--){
        int k=read_int();
        enter(f[4*k+1]-1-(f[4*k+1]-(4*k+1))/2);
    }
    return 0;
}
```

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