

# 结论 3

## 1、等差序列判定

给定一个序列  $a_1, a_2, \dots, a_n$  则  $(\max(a_i) - \min(a_i)) \geq (n-1) \times \text{gcd}(a_2 - a_1, a_3 - a_2, \dots, a_n - a_{n-1})$

等号成立充要条件为  $a_1, a_2, \dots, a_n$  从小到大排列后可以构成等差序列。

证明：设  $g = \text{gcd}(a_2 - a_1, a_3 - a_2, \dots, a_n - a_{n-1})$

由于  $g \mid a_{i+1} - a_i, a_{i+2} - a_{i+1}, \dots, a_j - a_{j-1}$  所以  $g \mid a_j - a_i$  得  $\text{gcd}(g, a_j - a_i) = g$

于是  $g = \text{gcd}(a_i - a_j) \ (1 \leq i, j \leq n)$  将  $a_i$  从小到大排序得到  $b_1, b_2, \dots, b_n$  则

$$g \leq \text{gcd}(b_2 - b_1, b_3 - b_2, \dots, b_n - b_{n-1}) \leq \min(b_2 - b_1, b_3 - b_2, \dots, b_n - b_{n-1}) \leq \frac{b_2 - b_1 + b_3 - b_2 + \dots + b_n - b_{n-1}}{n-1} = \frac{b_n - b_1}{n-1}$$

即  $g \leq \frac{\max(a_i) - \min(a_i)}{n-1}$  当  $a_i$  构成等差序列时有  $\frac{\max(a_i) - \min(a_i)}{n-1} \mid g$  此时有  $\frac{\max(a_i) - \min(a_i)}{n-1} = g$  证毕。

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Last update: 2021/07/21 09:35