由 F(x)F(x) 得到 F(x+c)F(x+c) □

F(x+c)=\sum\limits_{i=0}^nF[i](x+c)^iF(x+c)= i=0 ∑ n F[i](x+c) i

=\sum\limits_{i=0}^nF[i]\sum\limits_{j=0}^i\dbinom{i}{j}x^jc^{i-j}= i=0 ∑ n F[i] j=0 ∑ i (j i)x j c i−j

(二项式定理)

=\sum\limits_{i=0}^nF[i]\sum\limits_{j=0}^i\dfrac{i!}{j!(i-j)!}x^jc^{i-j}= i=0 ∑ n F[i] j=0 ∑ i

j!(i−j)! i! x j c i−j

=\sum\limits_{i=0}^nF[i]i!\sum\limits_{j=0}^i\dfrac{x^j}{j!}\dfrac{c^{i-j}}{(i-j)!}= i=0 ∑ n F[i]i! j=0 ∑ i

j! x j

(i−j)! c i−j

=\sum\limits_{j=0}^n\dfrac{x^j}{j!}\sum\limits_{i=j}^nF[i]i!\dfrac{c^{i-j}}{(i-j)!}= j=0 ∑ n

j! x j

i=j ∑ n F[i]i! (i−j)! c i−j

(交换和式)

设 G(x)=F(x+c)G(x)=F(x+c) ,提取系数可得：

G[j]=\dfrac{1}{j!}\sum\limits_{i=j}^nF[i]i!\dfrac{c^{i-j}}{(i-j)!}G[j]= j! 1

i=j ∑ n F[i]i! (i−j)! c i−j

我们设 P[n]=F[n]n!,\ H[n]=\dfrac{c^{n}}{n!}P[n]=F[n]n!, H[n]= n! c n

，则 G[j]j!=\sum\limits_{i=j}^nP[i]H[i-j]G[j]j!=

i=j ∑ n P[i]H[i−j] 这就是差卷积了。